Consistent Dyson summation of Higgs propagators in non-linear parameterizations revisited

C. Schwinn[a]

Institut für Physik, Johannes-Gutenberg-Universität, Staudingerweg 7, 55099 Mainz, Germany

Abstract. As we demonstrate in a process independent way, in a non-linear parameterization of the scalar sector of the standard model the Dyson summation of the Higgs self-energy can be performed without violating the Ward identities. This implies also the Goldstone boson equivalence theorem, in the limited range of its validity in effective field theories. This proves an earlier conjecture of Valencia and Willenbrock. Furthermore, the full Higgs propagator is independent of the gauge parameters. These results are consistent with the extension of the "gauge flip" formalism for the construction of gauge invariant classes of Feynman diagrams to loop diagrams. In a non-linear parameterization of a 2-Higgs doublet model, the consistent Dyson summation is possible for all neutral Higgs bosons, but not for the charged scalars. Explicit examples of the equivalence theorem are discussed both in the minimal standard model and a 2-Higgs doublet model.

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1 Introduction

Despite the phenomenological success of the electroweak standard model (SM), the underlying mechanism of electroweak symmetry breaking remains to be verified experimentally. Assuming this symmetry breaking is implemented in nature by the Higgs mechanism, one of the main goals of the next generation of collider experiments will be the study of the Higgs sector [1]. An extended Higgs sector is predicted for instance by the minimal supersymmetric standard model that involves a specific 2-Higgs doublet model (2HDM). The determination of the couplings of the Higgs bosons makes it necessary to consider processes with up to six or eight fermions in the final state for which a complete calculation of radiative corrections is currently not viable.

A useful tool to probe the symmetry breaking sector of the SM is the Goldstone boson equivalence theorem (ET) [2–5] that relates scattering amplitudes of longitudinally polarized massive gauge bosons to that of the associated Goldstone bosons (GBs). In the heavy Higgs limit, one can further replace internal gauge boson lines by GBs [6], allowing one to simplify higher order calculations of heavy Higgs effects [7].

Calculations of cross sections involving resonant Higgs bosons require a careful treatment of the Higgs resonance, especially since the width depends strongly on the Higgs mass. As is well known for massive gauge bosons, the violations of gauge invariance from inconsistent prescriptions for

finite width effects can lead to dramatic errors in the calculations of cross sections. It has been shown that fermionic self-energy contributions can be resummed consistently [8] if the fermionic corrections to irreducible vertices are evaluated at the same order of perturbation theory. For the Higgs self-energy, however, the fermionic contributions will be not sufficient in the mass region $m_H > 2m_W$ where decay into gauge bosons gives the dominant contribution to the Higgs width. A consistent treatment of the bosonic contributions is possible in the framework of the background field gauge [9] that requires a calculation of the complete radiative corrections at a fixed loop order which is presently not viable for the multi-fermion final states relevant for the measurement of the Higgs boson couplings. While a dependence on the quantum gauge parameter remains, in the Feynman gauge the background field gauge reproduces the results of the pinch technique [10]. Other suggested schemes for the treatment of unstable particles include the pole scheme [11] and the use of an effective Lagrangian including Wilson lines [12]. Recently an approach based on collinear effective field theory has been proposed [13] that has not yet been applied to realistic calculations. For suggested schemes for the treatment of the Higgs resonance see e.g. [14–18].

The compatibility of the Dyson summation of the Higgs propagator with the equivalence theorem has been discussed by Valencia and Willenbrock [14]. Since the ET holds order by order in perturbation theory while the Dyson summation mixes different orders, one cannot expect that the ET holds in a "naive" way when finite widths are

[a] e-mail: schwinn@thep.physik.uni-mainz.de

introduced. Indeed, as has been demonstrated in [14], a careful treatment of vertex corrections and non-resonant contributions is necessary to establish the ET in the Higgs resonance region. It was conjectured in [14] that the situation is simpler in a non-linear parameterization of the SM [19] and a "naive" version of the ET is satisfied i.e. the gauge boson and the Goldstone boson amplitude agree manifestly, also after introduction of a finite Higgs width and without taking vertex corrections into account. It is plausible that such a simplification occurs in a non-linear parameterization where the Higgs boson transforms trivially under gauge transformations and Becchi–Rouet–Stora–Tuytin (BRST) transformations, allowing to construct effective field theories without [19] or with a non-standard Higgs boson [20, 21]. Thus one can expect that Feynman diagrams involving Higgs bosons can be consistently treated separately from those without Higgs bosons. Indeed, using the "gauge flip" formalism for the construction of gauge invariant classes (GICs) of tree-level Feynman diagrams [22], a classification of GICs of tree-level diagrams in the non-linearly parameterized SM (NL-SM) in terms of the number of internal Higgs bosons has been given in [23].

In a non-linear parameterization, such a decomposition of the amplitude need not respect the good high energy behaviour implied by partial wave unitarity, despite being gauge invariant. Also the applicability of the ET is limited [5] compared to linear parameterizations. Nevertheless, disentangling the Higgs diagrams from the more involved gauge boson contributions and using the ET to work with the simpler GB amplitudes allows for a more transparent discussion of the unitarity violations induced by a finite Higgs width. This has been used in [15] to obtain a simple prescription for the Higgs propagator including a running width without violating unitarity in gauge boson scattering. Such a prescription is applicable in tree-level calculations and can be implemented in computer programs for the generation of scattering matrix elements. The prescription of [15] has also proven useful to obtain unitarity bounds on couplings of a non-standard Higgs boson [21].

In this note, we revisit the properties of Higgs propagators in non-linearly parameterized scalar sectors, providing a first example for the conjectured extension of the "flip" formalism to loop diagrams [22]. While a formal proof of this formalism on the one loop level will be given elsewhere [24, 25], in this note we use it as an intuitive tool and give direct proofs for the properties suggested by this formalism.

In Sect. 2 we review the formalism of flips and its extension to loop diagrams [24, 25]. Applied to the Higgs resonance in the NL-SM, the flip formalism is consistent with the conjecture of [14] and, in addition, with gauge parameter independence of the Higgs propagator. In Sect. 3 we discuss the ET in the NL-SM and the high energy behaviour of individual GICs. In Sect. 4 we give a formal proof and a one loop example for gauge parameter independence of the Higgs propagator in the NL-SM, including a discussion of H–Z mixing induced by CP-violation. In Sect. 5 we extend our analysis to a non-linear parameterization of the 2HDM [28]. We show that a consistent Dyson

summation can only be performed for the neutral Higgs bosons, including the CP-odd scalar while it is inconsistent for the charged Higgs bosons. We give an explicit example for the violation of the naive ET by the width of the charged Higgs bosons.

2 Flips, groves and the Higgs resonance

2.1 Gauge invariant classes of tree diagrams

Let us briefly review the formalism of [22] for the determination of GICs of tree diagrams and the application to non-linearly parameterized scalar sectors [23], before we describe its extension to loop diagrams and implications for the description of the Higgs resonance in Sect. 2.2. GICs are defined as subsets of Feynman diagrams contributing to a scattering matrix element that satisfy the appropriate Ward identities (WIs) by themselves and are gauge parameter independent. GICs can also be defined for Green's functions with off-shell particles [23] but the relevant definition involving the Slavnov–Taylor identities (STIs) is rather technical and will not be reproduced here.

In [22] it has been realized that the problem of constructing GICs of tree diagrams can be solved recursively by considering minimal sets of 4-point sub-diagrams that satisfy the appropriate WIs. In an unbroken gauge theory they are given by

$$G_{4,2F} = \left\{ \vcenter{\hbox{}} \right\}, \quad (1a)$$

$$G_4 = \left\{ \vcenter{\hbox{}} \right\}. \quad (1b)$$

The construction of GICs is based on defining elementary "gauge flips" as exchanges of diagrams in these sets. Gauge flips among larger diagrams are defined by applying elementary flips to sub-diagrams. For instance, the five point diagrams

$$(2)$$

are connected by elementary gauge flips, as denoted by a double arrow. This procedure of "flipping" gauge bosons through diagrams is in fact a formalization of the prescription to insert a gauge boson at all possible places into a diagram, familiar from the diagrammatic proof of the WI

in QED. It should be noted, however, that the gauge flips have to be applied both to external and internal gauge bosons present in Feynman diagrams. A set of diagrams like (2) that is closed under the application of flips – i.e. all diagrams in the set are connected by gauge flips and no diagram outside the set can be obtained by a gauge flip – is called a "grove". As was shown in [22], groves are the *minimal* GICs of tree diagrams. Examples for the structure of groves in the electroweak SM can be found in [22, 23].

To apply the flip formalism to the Higgs resonance, we need the correct form of the gauge flips in spontaneously broken gauge theories. If the scalar sector of the SM is parameterized linearly, it turns out that Higgs boson exchange diagrams have to be included in the gauge flips in addition to (1) while they can be omitted in a non-linear parameterization [23]. Therefore a flip

$$\tag{3}$$

is absent in the NL-SM but has to be included in a linear parameterization. Here and in the following, plain lines denote arbitrary particles. We take it as understood that in an R_ξ gauge the appropriate diagrams with internal GBs have to be included in addition. The complete list of elementary gauge flips in non-linear parameterizations of general Higgs sectors can be found in [23]. We will give some details on the formal reason for this simplification of the gauge flips at the end of this subsection. The absence of the flip (3) leads to the emergence of additional GICs in the NL-SM that can be classified according to the number of internal Higgs bosons [23]. This property will also be proven using a different formalism in Sect. 3 where it is used to proof the simplification of the ET observed in [14].

As an example for the decomposition of an amplitude into groves, consider the tree-level diagrams contributing to the process $e^+e^- \to \bar{b}bZ$ when the Higgs coupling to the electrons is set to zero. In the NL-SM they fall into three groves:

$$\tag{4a}$$

$$\tag{4b}$$

$$\tag{4c}$$

In a linear parameterization, there is a flip connecting the "Higgsstrahlung" diagram G_H to the diagrams in G_t, so only two groves remain.

In order to be able to obtain the gauge flips in the 2HDM in Sect. 5, we have to review the formal reason for the simplification of the gauge flips in non-linear parameterizations. In a spontaneous broken gauge theory in R_ξ gauge, the diagrams that have to be taken into account in the definition of the elementary gauge flips are determined by the requirement that all corresponding four point diagrams with external GBs also satisfy the appropriate WIs [23]. The tree-level Higgs interaction vertex functions in the NL-SM satisfy the simple WIs (here $V_a = W^\pm, Z$ and ϕ_a are the associated GBs)

$$\mathrm{i}p_{a\mu}\Gamma^{\mu\nu}_{V_a V_b H} + m_{V_a}\Gamma^{\nu}_{\phi_a V_b H} = 0 \,, \tag{5a}$$

$$\mathrm{i}p_{a\mu}\Gamma^{\mu}_{V_a \phi_b H} + m_{V_a}\Gamma_{\phi_a \phi_b H} = 0 \,. \tag{5b}$$

For a formal derivation of these identities and our notation used for vertex functions see Appendix A.1. Since the identities (5) do not require the Higgs to be on shell, they imply that the Higgs exchange diagram in (3) and the corresponding diagram with one external GB satisfy a WI by themselves. Therefore the gauge flip (3) can be omitted in the NL-SM. In a linear parameterization, there are additional terms contributing to (5) and the flip (3) cannot be omitted.

## 2.2 Groves of loop diagrams and application to the Higgs resonance

The action of gauge flips can be extended to loop diagrams [24], taking into account that the external legs of 4-point sub-diagrams can be connected to form a closed loop. A computer program, mangroves, for the determination of groves of loop diagrams is currently in preparation [24]. It is plausible to conjecture [22] that the groves obtained in this way are the minimal GICs of loop diagrams, since one does not expect that for loop diagrams a finer partitioning of the amplitude into GICs will be possible than at tree level. Here we do not attempt a formal proof [25] but take the attitude that groves are sensible candidates for minimal GICs of loop diagrams. As an example of a grove of

loop diagrams, consider the one loop quark gluon vertex in QCD where the application of the flips (1) results in the set of diagrams (we take it as understood that the appropriate ghost contributions have to be included in addition)


$$\tag{6}$$
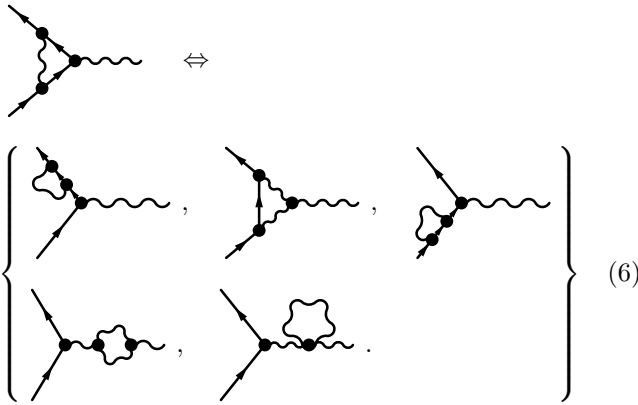
The diagrams in (6) should be considered as part of a larger diagram, so in general it will be necessary to "flip" the gauge boson lines further through the complete diagram. Since there is no flip connecting the vertex correction to the fermion-loop diagrams contributing to the vacuum polarization, these diagrams form a separate grove. Therefore in this example the flip formalism is consistent with the fermion-loop scheme [8].

Applying the flip formalism to the Higgs resonance, we immediately find that the Higgs self-energy and the vertex corrections are not connected by gauge flips in the NL-SM because the Higgs exchange diagrams are not included in the elementary gauge flips (3):

$$\tag{7}$$
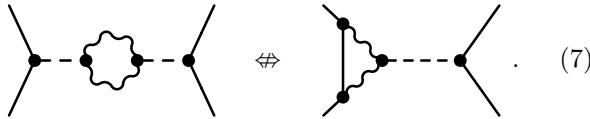
This property is independent of the external particles attached to the Higgs propagators. Similarly, there are no flips to irreducible higher order contributions to the self-energy:

$$\tag{8}$$
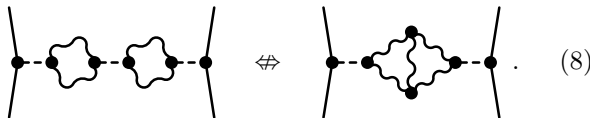
Provided the identification of groves with minimal GICs holds for loop diagrams, the results of the flip formalism therefore indicate that a resummation of the self-energy insertions without including vertex corrections or higher order contributions to the self-energy does not violate WIs or gauge parameter independence. Since the ET is a consequence of the WIs and the kinematical properties of the longitudinal gauge boson polarization vector, this is consistent with the conjecture of [14]. In contrast, the gauge flip (3) has to be included in a linear parameterization so that vertex corrections and irreducible contributions to the self-energy consistently have to be considered at the

same loop order. The gauge parameter independence of the Higgs propagator has not been discussed in [14] and will be treated in more detail in Sect. 4.

3 Goldstone boson equivalence theorem and the Higgs resonance

Using the formalism of gauge flips, we have motivated that the Dyson summation of the Higgs resonance in the NL-SM does not violate WIs or the Goldstone boson equivalence theorem, in agreement with the conjecture of [14]. Before providing a general proof in Sect. 3.2, we give an explicit example for the ET in the NL-SM and discuss the high energy behaviour of GICs in non-linear parameterizations.

3.1 Equivalence theorem and unitarity: an example

As example for the ET in a non-linear parameterization, we consider top production by vector boson fusion [27], following the discussion of vector boson scattering in [14, 15]. This example has also been discussed in [4] for the heavy Higgs limit in a linear parameterization, and an effective field theory analysis of effects of a non-standard Higgs boson has been given in [21]. According to the flip formalism, in the NL-SM there are two separately gauge invariant sets of diagrams

$$\tag{9}$$

As we will now show, the ET holds separately for both classes of diagrams. As a caveat, the additional GICs in the NL-SM do not necessarily have a good high energy behavior taken by themselves since in non-linear parameterizations tree-level unitarity is not a consequence of gauge invariance. According to [26], the only theories of massive vector bosons respecting tree-level unitarity – i.e. the requirement that the tree-level matrix elements for N-particle scattering amplitudes at high energies scale at most as E^{4-N} – are equivalent to linearly parameterized spontaneously broken gauge theories. In the NL-SM with standard Higgs couplings, reparameterization invariance of the S matrix implies that the *complete* set of diagrams satisfies the tree-level unitarity bound, so the classes of diagrams that show good high energy behavior will in general be the same in both parameterizations. This will become apparent in the example below.

In the NL-SM, the Higgs–gauge boson and Higgs–GB vertices arise from the operator

$$\frac{1}{2}vH\mathrm{tr}[(D_\mu U)^\dagger D^\mu U] \qquad (10)$$

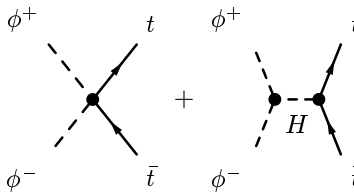$$= \frac{g}{m_W}H(\partial^\mu\phi^+ + m_W W^{+,\mu})(\partial_\mu\phi^- + m_W W^-_\mu) + \cdots$$

that includes also a similar term involving the Z and ϕ^0 bosons and terms of higher order of the GBs not shown here. Here we have defined $U = \exp\left(\mathrm{i}\frac{\phi\cdot\sigma}{v}\right)$ and the covariant derivative is given by $D_\mu U = \partial_\mu U + \mathrm{i}g W^i_\mu \frac{\sigma^i}{2}U - \mathrm{i}g' U B \frac{\sigma^3}{2}$. It can be checked that the vertices obtained from (10) satisfy the WIs (5). Since the operator (10) is gauge invariant by itself, in an effective field theory approach to a non-standard Higgs [20], it can be included with an arbitrary coefficient. The Yukawa couplings arise from the operators

$$\mathcal{L}_Y = -\bar{Q}_L U\left[\frac{(m_t + m_b)}{2} + \frac{(m_t - m_b)}{2}\sigma^3\right]Q_R \qquad (11)$$

$$-H\bar{Q}_L U\left[\frac{(\lambda_t + \lambda_b)}{2} + \frac{(\lambda_t - \lambda_b)}{2}\sigma^3\right]Q_R + \mathrm{h.c.},$$

where gauge invariance allows for Higgs–Yukawa couplings $\lambda_{t,b}$ not related to the fermion masses $m_{t,b}$. Neglecting the bottom mass, the relevant interaction terms are given by

$$\mathcal{L}_Y = -\frac{\mathrm{i}m_t}{\sqrt{2}v}\phi^+\bar{t}\left(\frac{1-\gamma^5}{2}\right)b + \frac{\mathrm{i}m_t}{\sqrt{2}v}\phi^-\bar{b}\left(\frac{1+\gamma^5}{2}\right)t$$

$$-\lambda_t H\bar{t}t + \frac{m_t}{v^2}\phi^+\phi^-\bar{t}t + \cdots. \qquad (12)$$

We will use the ET to calculate the contributions that grow with the energy and potentially can violate the unitarity bound, i.e. we calculate the diagrams for $\phi^+\phi^- \to t\bar{t}$ in the limit $m_W \to 0$ with $v = \frac{2m_W}{g} = \mathrm{const.}$ while keeping the top mass fixed. While the t-channel diagram shows no dangerous high energy behavior, the Higgs exchange diagram and the contact diagram from the interaction quadratic in the GBs in (12) grow linearly with the energy[1]:

$$= \mathrm{i}\left(\frac{m_t}{v^2}\right)\bar{t}t\left[1 - \left(\frac{\lambda_t v}{m_t}\right)\frac{s}{s - m_H^2 + \mathrm{i}\,\mathrm{Im}\,\Pi_H(s)}\right] \qquad (13)$$

where we have used the Dyson summation to introduce the imaginary part of the Higgs self-energy Π_H into the propagator. As can be checked using the Feynman rules obtained from (10), the Higgs exchange diagram is reproduced by the corresponding diagram with external longitudinal W

bosons in the limit where the gauge boson mass is sent to zero. The contact diagram is reproduced from the t-channel diagram and the s-channel Z/γ exchange diagrams after a non-trivial cancellation of terms growing like E^2 [4]. Therefore this diagram is connected to the grove G_g in (9) by the ET. This shows that, in agreement with the expectation from the flip formalism, in the NL-SM the ET holds separately for the groves G_g and G_H in (9), also for a finite width. However, to obtain good high energy behavior, all diagrams from both G_g and G_H have to be considered. Furthermore, only for the SM value of the Higgs–Yukawa coupling $\lambda_t = \frac{m_t}{v}$ and a vanishing width both diagrams add up to an amplitude with good high energy behavior proportional to $\frac{m_H^2}{s - m_H^2}$. In an effective field theory approach, unitarity up to the cutoff of the effective theory implies bounds on the non-standard couplings [21] like λ_t.

In a linear parameterization, the contact diagram appearing in (13) is absent while in the numerator of the Higgs exchange diagram s is replaced by m_H^2 so for a vanishing width the same result for the GB amplitude is obtained as in the NL-SM *without cancellation among diagrams*. However, the GB amplitude agrees with the one for the longitudinal gauge bosons only for a vanishing width, so the Dyson summation is incompatible with the naive ET in the linear parameterization.

Since the imaginary part of the gauge boson contributions to the Higgs self-energy is proportional to s^2 [15], the result (13) violates the unitarity bound when a realistic expression for Π_H is inserted. To include a running width without violating unitarity in the NL-SM, a modified Higgs propagator [15]

$$D_H(q^2) \to \frac{\mathrm{i}(1 + \mathrm{i}\gamma_H)}{q^2 - m_H^2 + \mathrm{i}\gamma_H q^2} \quad \text{with} \quad \gamma_H = \Gamma_H/M_H\theta(q^2) \qquad (14)$$

has been proposed in the context of gauge boson scattering (for a generalization beyond leading order see [18]). For the SM value of λ_t, using this form of the propagator allows for the cancellation between the Higgs exchange diagram and the contact diagram to take place in (13) and one obtains an amplitude proportional to $\frac{m_H^2}{s(1+\mathrm{i}\gamma_H) - m_H^2}$. Therefore good high energy behavior is restored also for a running width. This has to be compared to the case of the W boson, where the introduction of a running width [29] similar to (14) is in general incompatible with gauge invariance unless all radiative corrections are included in the same order, resulting in possibly large numerical errors in certain regions of phase space [8]. In a linear parameterization, also the propagator (14) is not compatible with the naive ET.

To summarize, in a non-linear parameterization there are additional GICs compared to a linear one that satisfy the ET by themselves, also for a finite width of the Higgs boson. However, the classes of diagrams that are gauge invariant *and* show good high energy behavior will in general be the same in both parameterizations.

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[1] Recall that the spinors t scale with $\sqrt{E}$.

3.2 Compatibility of the ET with Dyson summation: general discussion

After this example, we proceed to a general proof of the consistency of the Dyson summation of the Higgs propagator in the NL-SM and discuss the relation to the conjectured simplification of the ET [14] in more detail. To be precise, in [14] it has been suggested that matrix elements can contain external Higgs bosons off their mass shell without violating the ET, provided a non-linear parameterization is used. This property is in fact a straightforward consequence of the trivial BRST transformation $\delta_{\mathrm{BRST}} H = 0$ of the Higgs in the NL-SM. Recall that the derivation of the ET requires the WIs for amplitudes with insertions of the operator $(\mathrm{i} p_\mu V_a^\mu + m_{V_a} \phi_a)$ [2]. In the BRST formalism, they are derived using the Kugo–Ojima condition that the BRST charge Q annihilates physical states. This results in STIs of the form

$$0 = \langle \mathrm{out} | T \left[\{Q, \bar{c}_a B_b \dots B_n\} \right] | \mathrm{in} \rangle$$
$$= \langle \mathrm{out} | T \left[B_a \dots B_n \right] | \mathrm{in} \rangle \, , \qquad (15)$$

where B_a is the Nakanishi–Lautrup auxiliary field obtained from the BRST transformation of an antighost $\bar{c}_a$. We use a linear R_ξ gauge fixing also in the non-linear parameterization, so that the equation of motion of B_a is given by $B_a = -\frac{1}{\xi}(\partial_\mu V_a - \xi m_{V_a} \phi_a)$ as in the linear parameterization. In the NL-SM, the trivial BRST transformation of the Higgs implies that similar identities are true also if additional Higgs boson field operators are inserted in the Green's functions:

$$0 = \langle \mathrm{out} | T \left[\{Q, \bar{c}_a B_b \dots B_n H \dots H\} \right] | \mathrm{in} \rangle$$
$$= \langle \mathrm{out} | T \left[B_a \dots B_n H \dots H \right] | \mathrm{in} \rangle \, . \qquad (16)$$

In a linear parameterization, the BRST transformation $\delta_{\mathrm{BRST}} H = \frac{g}{2}(c^+ \phi^- + c^- \phi^+) + \frac{g}{2\cos\theta_{\mathrm{W}}} c_Z \phi_0$ mixes the Higgs with the GBs so there are additional terms on the right hand side of (16).

The WIs (16) allow in the usual way [2,5] to deduce the validity of the ET also for matrix elements with external off-shell Higgs bosons $H^\star$, in agreement with the conjecture of [14]:

$$\mathcal{M}(\mathrm{in} \to \mathrm{out} + V_a^L \dots V_n^L H^* \dots H^*)$$
$$= (-\mathrm{i})^n \mathcal{M}(\mathrm{in} \to \mathrm{out} + \phi_a \dots \phi_n H^* \dots H^*)$$
$$+ \mathcal{O}\left(\frac{m_V}{E} - \mathrm{suppressed}\right) \, , \qquad (17)$$

where one phase of $(-\mathrm{i})$ occurs for every outgoing longitudinal gauge boson V^L (for incoming gauge bosons, the sign of the phase has to be changed) and we have suppressed renormalization factors [3]. While in a renormalizable theory and for external on-shell Higgs bosons, the additional contributions on the right hand side are of order $O\left(\frac{m_V}{E}\right)$ and the GB amplitude is bounded at large energies, in a non-linearly parameterized effective field theory, the additional contributions are suppressed only relative to the GB amplitude [5] that need not show good high energy

behaviour. To assess the usefulness of the ET in a given situation, the absolute value of the additional terms has to be estimated for the process and the energy range of interest [5]. We always take it as understood that the ET in a non-linearly parameterized theory holds in this restricted sense.

We now demonstrate that (17) already implies also the validity of the ET for internal off-shell Higgs bosons, also when a Dyson resummed propagator is used. To show this, we give a diagrammatic prescription to express the complete amplitude in terms of matrix elements with off-shell Higgs bosons and all other particles on the mass shell. Consider the set of diagrams that has in common an internal Higgs boson line (that is not part of a closed loop) with a given momentum p_H, labeling the external momenta such that $-(p_1 + \dots + p_i) = p_H = p_{i+1} + \dots + p_n$ and treating all momenta as incoming. This set of diagrams can be written in the factorized form

$$= \mathcal{M}(\Psi_1 \dots \Psi_i H^*) \, D_H(p_H) \, \mathcal{M}(H^* \Psi_{i+1} \dots \Psi_n) \quad (18)$$

We have depicted the case of a s-channel Higgs boson, but a similar decomposition holds for t- or u-channel Higgs lines. If the complete amplitude is evaluated at a given loop order, the decomposition (18) has to be understood as consistently expanded up to this order. Iterating the decomposition (18) by applying the same formula to the sub-amplitudes[2] we arrive at a decomposition of the amplitude in terms of matrix elements with off-shell Higgs bosons and all other external particles on the mass shell. When applied to arbitrary internal particles off the mass shell, the individual terms of such a decomposition are in general not gauge invariant by themselves. For the case of Higgs bosons in the NL-SM, however, the sub-amplitudes occurring in (18) are precisely the quantities satisfying the ET for off-shell Higgs bosons (17). As a consequence, the contributions to the S-matrix with internal Higgs boson lines with a given set of momenta satisfy the ET by themselves. Since this property is independent of the expression used for the Higgs propagators D_H in (18), we can use the Dyson resummed propagator or a simple effective prescription like (14) without violating WIs or the ET. As in the example of Sect. 3.1, the subsets of diagrams with internal Higgs bosons need not respect unitarity bounds. Nevertheless, the separate gauge invariance can be useful for the discussion of simple schemes to restore unitarity.

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[2] There is a subtlety in avoiding double counting of diagrams. For instance, first applying (18) to a Higgs boson line with some momentum p_{H_1} and subsequently factorizing another momentum p_{H_2} out of a sub-amplitude yields contributions that appear also when the contribution of p_{H_2} is factorized first. Such contributions generated more than once have to be omitted.

In addition to the simple WIs (16), the trivial BRST transformation law of the Higgs boson implies the gauge parameter independence of Green's functions with off-shell Higgs bosons, if all other external particles are on shell. As reviewed briefly in Appendix A.2, in the BRST formalism the gauge parameter dependence of Green's functions can be expressed in terms of Green's functions with insertions of the BRST transformed fields, cf. (66). From this, one obtains immediately

$$\partial_\xi \langle \text{out}| T[H\dots H]|\text{in}\rangle = 0\,. \tag{19}$$

By the same reasoning as above, this implies gauge parameter independence of subsets of diagrams with a given set of momenta of internal Higgs lines, independent of the prescription used for the Higgs propagator (provided it is gauge parameter independent by itself). Incidently, these results give an independent derivation of the classification of GICs in the NL-SM in terms of the number of internal Higgs boson lines derived in [23] using the flip formalism.

4 Properties of the Higgs self-energy in the non-linear parameterization

Apart from the consistency of the Dyson summation with the ET, another result motivated by the flip formalism in Sect. 2 is the gauge parameter independence of the Higgs propagator in the NL-SM:

$$\partial_\xi \langle 0|T[H(x)H(y)]|0\rangle = 0\,. \tag{20}$$

In fact, this is merely a special case of the gauge parameter independence of matrix elements with off-shell Higgs bosons (19). In a linear parameterization, (20) is violated because the BRST transformation of the Higgs field is non-trivial (see (66) and the remarks below (16)). In the presence of CP-violating mixing with the gauge boson sector, the full Higgs propagator includes contributions from the Z and GB propagators [30]:

$$\langle 0|T[H(x)H(y)]|0\rangle = \text{---}\otimes\text{---} + \text{---}\otimes\text{---}\otimes\text{---}$$
$$\qquad\qquad\qquad\qquad\quad \Gamma_{HH} \qquad\quad \Gamma_{HH}\ \Gamma_{HH}$$

$$+ \text{---}\otimes\text{\Large\textasciitilde}\otimes\text{---} + \text{---}\otimes\text{---}\otimes\text{---} + \dots\,. \tag{21}$$
$$\quad\ \Gamma_{HZ}\ \ \Gamma_{ZH} \qquad\quad \Gamma_{H\phi^0}\ \Gamma_{\phi^0 H}$$

Therefore the gauge parameter independence of the full propagator (20) will in general be a consequence of cancellations among the different contributions, and the Higgs self-energy by itself can be gauge parameter dependent. This will be discussed from the perspective of the flip formalism below.

But first, let us demonstrate explicitly the cancellation of the gauge parameter in the one loop gauge boson contribution to the Higgs propagator. We decompose the gauge boson propagator in R_ξ gauge into the propagator in unitarity gauge and a term proportional to the GB propagator, introducing the graphical notation

$$\sim\!\!\sim\!\!\sim = \text{\Large QQQQ} \qquad\qquad + \quad \bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet$$

$$D_{W,\xi}^{\mu\nu}(q) = \frac{-\mathrm{i}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_W^2}\right)}{q^2 - m_W^2} \quad + \left(-\frac{q^\mu q^\nu}{m_W^2}\right)\frac{\mathrm{i}}{q^2 - \xi m_W^2}\,. \tag{22}$$

Because of the trivial BRST transformation, there are no ghost–Higgs vertices so we only have to consider the gauge boson and GB loops. The peculiar form of the interaction lagrangian (10) ensures that the contributions with one unphysical pole add up to zero:

$$\begin{aligned}
&\overset{p+k}{\underset{k}{\text{---}\bullet\!\text{QQ}\!\bullet\text{---}}} = (\mathrm{i}gm_W)^2\int \mathrm{d}^4k\,\frac{-\mathrm{i}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{m_W^2}\right)}{k^2 - m_W^2}\\
&\qquad\qquad \times\left(-\mathrm{i}\frac{(p+k)_\mu(p+k)_\nu}{m_W^2((p+k)^2 - \xi m_W^2)}\right),
\end{aligned} \tag{23a}$$

$$\begin{aligned}
&\overset{p+k}{\underset{k}{\text{---}\bullet\!\text{QQ}\!\bullet\text{---}}} = g^2\int \mathrm{d}^4k\,(p+k)_\mu\,\frac{-\mathrm{i}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{m_W^2}\right)}{k^2 - m_W^2}\\
&\qquad\qquad \times\frac{\mathrm{i}}{(p+k)^2 - \xi m_W^2}\left(-(p+k)_\nu\right).
\end{aligned} \tag{23b}$$

In the second diagram, one minus sign arises because the gauge boson momentum is incoming at one vertex and outgoing at the other. One can show similarly that the contributions with two unphysical poles add up to zero. In contrast, in the linear parameterization the $H\Phi W$ vertex has the form $\frac{g}{2}W^{\pm,\mu}H\overset{\leftrightarrow}{\partial_\mu}\phi^\mp$ and the cancellation does not go through as in (23), so the gauge boson contribution is gauge parameter dependent [17].

In the presence of Higgs–Z mixing, gauge parameter independence of the Higgs propagator does not already imply gauge parameter independence of the Higgs self-energy to all orders. As shown in Appendix A.2 using the formalism of [31,32], in the presence of CP-violation the gauge parameter dependence of the Higgs self-energy takes the form

$$\partial_\xi \Gamma_{HH} = 2\Lambda_{\phi^0 H}\Gamma_{\phi^0 H} + 2\Lambda_{ZH,\mu}\Gamma_{ZH}^\mu\,, \tag{24}$$

where $\Lambda_{\phi^0 H}$ and Λ_{ZH} are vertex functions with insertions of the gauge parameter dependent part of the gauge fixing functional (see Appendix A.2 for the precise expressions). In the SM this mixing is phenomenologically not important since it is induced by CP-violating effects only at the three loop level, but the phenomenon will persist in CP-violating extensions of the SM. The discussion is also relevant for the CP-odd scalar A in the 2HDM discussed in Sect. 5. The reason why CP-violating Higgs–Z mixing introduces additional complications can also be understood directly on the level of gauge flips. Once ZH mixing is generated radiatively as in the minimal SM by an insertion of a box

diagram [30], there is a gauge flip to vertex correction diagrams:

Since the full resummed Higgs propagator (21) includes also contributions from Higgs–Z mixing, similar flips connect the irreducible Higgs self-energy to reducible diagrams with Higgs–Z mixing (compare to (8)), so once CP-violation occurs, the Higgs self-energy is not expected to be gauge parameter independent by itself. Nevertheless, the gauge parameter dependence must cancel between the different irreducible two point functions since the full Higgs propagator is gauge parameter independent.

To clarify this issue further, one can demonstrate the cancellation among the various contributions by applying the formalism of [31,32] to the full propagator (21) including mixing. The complete treatment to all orders involves a three by three matrix describing the mixing among Z, ϕ^0 and H and is beyond the scope of this note. Here we give a simplified analysis valid in the first loop order n where HZ mixing is non-vanishing. In this case we can restrict ourselves to the diagrams shown in (21) and need not consider the mixing of the Z boson with the GB ϕ^0. The variation of the Higgs propagator with respect to the gauge parameter receives contributions from the gauge boson and GB propagators and from the self-energies themselves:

$$
\partial_\xi D_H^{(2n)} = D_H^{(0)} \Big[\partial_\xi \Gamma_{HH}^{(2n)} + 2 \left(\partial_\xi \Gamma_{HZ}^{(n),\mu} \right) D_{Z,\mu\nu}^{(0)} \Gamma_{ZH}^{(n),\nu}
$$
$$
+ 2 \left(\partial_\xi \Gamma_{H\phi^0}^{(n)} \right) D_{\phi^0}^{(0)} \Gamma_{\phi H}^{(n)} \Big] D_H^{(0)}
$$
$$
+ D_H^{(0)} \Big[\Gamma_{HZ}^{(n),\mu} (\partial_\xi D_{Z,\mu\nu}^{(0)}) \Gamma_{ZH}^{(n),\nu}
$$
$$
+ \Gamma_{H\phi^0}^{(n)} (\partial_\xi D_{\phi^0}^{(0)}) \Gamma_{\phi^0 H}^{(n)} \Big] D_H^{(0)} . \quad (26)
$$

Here $D_H^{(2n)}$ denotes the Higgs propagator up to order $2n$, where n is first order where Γ_{HZ} is non-vanishing. In the order considered, the propagators of the Z and ϕ^0 are tree-level propagators so we can use the explicit expression (22) to verify that the terms involving the variation of the propagators cancel among themselves because of the simple WI $(ip_\mu \Gamma_Z^\mu + m_Z \Gamma_{\phi^0 H}) = 0$. To simplify the remaining terms we use the identity (64), making the plausible assumption that the vertex functions Λ_{HZ} and $\Lambda_{H\phi^0}$ arise only at the same order as the mixing Γ_{ZH}. Up to this order the two point vertex functions enter only on tree level and we obtain

$$
\partial_\xi \Gamma_{HZ}^{(n),\mu} = \Lambda_{HZ,\nu}^{(n)} \Gamma_{ZZ}^{(0),\nu\mu} \quad \partial_\xi \Gamma_{H\phi^0}^{(n)} = \Lambda_{H\phi^0}^{(n)} \Gamma_{\phi^0\phi^0}^{(0)} . \quad (27)
$$

Since the two point vertex functions are the negative of the inverse propagators, the gauge parameter dependence of the mixing contributions cancels against the variation of the Higgs self-energy (24):

$$
\partial_\xi D_H^{(2n)}
$$
$$
= D_H^{(0)} \Big[\partial_\xi \Gamma_{HH}^{(2n)} - 2 \left(\Lambda_{HZ,\mu}^{(n)} \Gamma_{ZH}^{(n),\mu} + \Lambda_{H\phi^0}^{(n)} \Gamma_{\phi H}^{(n)} \right) \Big] D_H^{(0)}
$$
$$
= 0 . \quad (28)
$$

Therefore the gauge parameter dependence of the full propagator vanishes, in agreement with (20). To establish this property, it is necessary to calculate the Higgs self-energy up to the order $2n$; a residual gauge dependence remains if it is evaluated at the same order as the HZ mixing.

5 Two-Higgs doublet models

As an important example for a non-minimal Higgs sector, in this section we discuss to which extent the results obtained for the NL-SM carry over to a 2-Higgs doublet model. A complication compared to the NL-SM is the appearance of vertices involving two Higgs bosons and a gauge boson like $W^{+,\mu}(H^0 \overleftrightarrow{\partial_\mu} H^-)$ that can lead to a more complicated structure of GICs [23]. As we will demonstrate below, in the 2HDM only the neutral Higgs bosons can be treated as in the NL-SM and their Dyson summation does not violate gauge invariance. Our discussion is mainly phrased in the language of gauge flips, but the formal proofs of Sects. 3 and 4 can easily be extended to the case of the neutral Higgs bosons in the 2HDM, since the main ingredient is the trivial BRST transformation of the neutral Higgs. We also give an explicit example for the violation of the naive ET by the introduction of a finite width for the charged Higgs boson.

5.1 Non-linear parameterization of the 2HDM

We will briefly review the 2HDM in the non-linear parameterization introduced in [28] and determine the BRST transformations of the Higgs bosons that are used in the subsequent subsections to derive the STIs and the form of the gauge flips. Following [28] we collect both scalar doublets H_1, H_2 of the 2HDM in a matrix

$$
\Phi = \left(\tilde{H}_2 H_1 \right) \quad \text{with} \quad \tilde{H}_i = i\sigma_2 H_i^*
$$
$$
\text{and} \quad \langle H_{1,2} \rangle = \begin{pmatrix} 0 \\ v_{1,2} \end{pmatrix} \quad (29)
$$

and introduce the non-linear parameterization $\Phi = U\mathcal{H}$, where the GB matrix is again $U = \exp\left(i\frac{\phi \cdot \sigma}{v} \right)$ and the Higgs bosons are collected in the matrix

$$
\mathcal{H} = \mathcal{H}_0 + \left(h + iA^0 + \sigma \cdot \mathbf{H} \right) \begin{pmatrix} \cos\beta & 0 \\ 0 & \sin\beta \end{pmatrix} \quad (30)
$$

$$\text{with} \quad \mathcal{H}_0 = \begin{pmatrix} v_2 & 0 \\ 0 & v_1 \end{pmatrix} . \tag{31}$$

Here the mixing angle $\tan \beta = \frac{v_2}{v_1}$ has been introduced and we define v so that $v_1 = v \cos \beta$ and $v_2 = v \sin \beta$. The mass eigenstates of the neutral scalars are linear combinations of h and H^3 but the precise form [28] is not needed for our discussion. More important for us are the "interaction eigenstates" [28] H and S defined by

$$\cos \beta S + \sin \beta H = \cos \beta \left(\frac{h + H^3}{2} \right) , \tag{32}$$

$$-\sin \beta S + \cos \beta H = \sin \beta \left(\frac{h - H^3}{2} \right) , \tag{33}$$

that will simplify the Feynman rules. The GB and Higgs matrices transform under $SU(2)_L \times U(1)_Y$ as

$$\Phi \to L\Phi R^\dagger \qquad U \to LUR^\dagger \qquad \mathcal{H} \to R\mathcal{H}R^\dagger \tag{34}$$

with $L = \mathrm{e}^{\mathrm{i}\alpha \cdot \frac{\sigma}{2}} \in SU(2)_L$ and $R = \mathrm{e}^{\mathrm{i}\beta \frac{\sigma^3}{2}} \in U(1)_Y$. Therefore the BRST transformation of the Higgs bosons is given by

$$\delta_{\mathrm{BRST}} \mathcal{H} = \mathrm{i}g'c^3 \left[\frac{\sigma^3}{2}, \mathcal{H} \right] , \tag{35}$$

so the BRST transformations of the charged Higgs bosons $H^\pm = \frac{1}{\sqrt{2}}(H^1 \mp \mathrm{i}H^2)$ are found to be

$$\delta_{\mathrm{BRST}} H^\pm = \pm \mathrm{i} \left(ec_A - \frac{g}{\cos \theta_{\mathrm{W}}} \sin^2 \theta_{\mathrm{W}} c_Z \right) H^\pm , \tag{36}$$

while the neutral Higgs bosons transform trivially. Below, we need the Feynman rules of the Higgs bosons appearing in the decomposition of the kinetic term

$$\mathcal{L}_{\mathrm{kin}} = \frac{1}{4} \mathrm{tr} \left[(D_\mu \Phi)^\dagger D^\mu \Phi \right] := \mathcal{L}_H + \mathcal{L}_U + \mathcal{L}_{HU} , \tag{37}$$

where we have defined the operators

$$\mathcal{L}_H = \frac{1}{4} \mathrm{tr} \left[D_\mu \mathcal{H}^\dagger D^\mu \mathcal{H} \right] , \tag{38a}$$

$$\mathcal{L}_U = \frac{1}{4} \mathrm{tr} \left[\left(D^\mu U^\dagger \right) (D_\mu U) \, \mathcal{H}^\dagger \mathcal{H} \right] , \tag{38b}$$

$$\mathcal{L}_{HU} = \frac{1}{4} \left(\mathrm{tr} \left[\left(U^\dagger D^\mu U \right) \left(\mathcal{H} D_\mu \mathcal{H}^\dagger \right) \right] + \mathrm{h.c.} \right) . \tag{38c}$$

Here the transformation law (34) implies the action of the covariant derivative on $\mathcal{H}$ as $D_\mu \mathcal{H} = \partial_\mu \mathcal{H} - \mathrm{i}g' B_\mu \left[\frac{\sigma^3}{2}, \mathcal{H} \right]$. Note that the operators in (38) are gauge invariant by themselves. Therefore, in an effective field theory approach to the 2HDM, the operator $\mathcal{L}_{HU}$ can appear with an arbitrary coefficient λ_{HU} while the coefficients of the other two operators are fixed by the normalization of the kinetic terms[3].

---

[3] For simplicity, here we do not consider the introduction of another kinetic term for the GBs of the form $\mathrm{tr}[(D_\mu U)^\dagger D^\mu U]$ that would allow for a non-standard coefficient of $\mathcal{L}_U$.

The cubic interaction terms obtained from the expansion of (38) will be written as

$$\mathcal{L}_{\Phi^3} = \sum_{\Phi_i \Phi_j \Phi_k} \left(\mathcal{O}^H_{\Phi_i \Phi_j \Phi_k} + \mathcal{O}^U_{\Phi_i \Phi_j \Phi_k} + \lambda_{HU} \mathcal{O}^{HU}_{\Phi_i \Phi_j \Phi_k} \right) , \tag{39}$$

where the fields Φ include the gauge bosons, Higgs bosons and GBs. Here the operators $\mathcal{O}^H_{\Phi_i \Phi_j \Phi_k}$ arise from the expansion of $\mathcal{L}_H$ and analogously for $\mathcal{O}^U_{\Phi_i \Phi_j \Phi_k}$ and $\mathcal{O}^{HU}_{\Phi_i \Phi_j \Phi_k}$.

5.2 Neutral Higgs bosons

We now discuss the form of the gauge flips involving neutral Higgs bosons and the consequences for the GICs and the Dyson summation of the Higgs propagators. The charged Higgs bosons are discussed in Sect. 5.3. The discussion of the neutral scalars is simpler in terms of the two interaction eigenstates defined in (32). As can be verified from the Feynman rules arising from $\mathcal{L}_U$, the interaction eigenstate H has the same interactions with the gauge bosons as the Higgs in the NL-SM (10), while all other scalars have no interactions of the form $HV_\mu V^\mu$ [28]. In the language of the flip formalism, this implies that the flips of the form (3) have to be chosen just like in the NL-SM, and the internal Higgs boson H can be omitted.

We now turn to the additional HHV vertices in the 2HDM. One finds that the only vertices of this kind involving neutral Higgs bosons are contained in the operator $\mathcal{L}_{HU}$ defined in (38) and involve only the second interaction eigenstate S and the CP-odd scalar A^0:

$$\mathcal{O}^{HU}_{W^+ H^- H^0} := \frac{1}{\sqrt{2}v} \left(\partial^\mu \phi_+ + m_W W^{+,\mu} \right) \tag{40}$$

$$\times \left[\frac{1}{2} \left(A^0 \overset{\leftrightarrow}{\partial_\mu} H^- \right) + \mathrm{i} \left(S \overset{\leftrightarrow}{\partial_\mu} H^- \right) \right] + \mathrm{h.c.}$$

$$\mathcal{O}^{HU}_{ZSA^0} := \frac{1}{v} \left(\partial^\mu \phi_0 + m_Z Z^\mu \right) \left(S \overset{\leftrightarrow}{\partial_\mu} A^0 \right) , \tag{41}$$

with $\phi_1 \overset{\leftrightarrow}{\partial_\mu} \phi_2 = \phi_1 \partial_\mu \phi_2 - (\partial_\mu \phi_1) \phi_2$. As a consequence of the trivial BRST transformations, one obtains trivial tree-level WIs as long as a neutral Higgs is involved:

$$\mathrm{i}p_{W,\mu} \Gamma^\mu_{W^\pm H^\mp S}(p_W, k_\mp, k_S)$$
$$+ m_W \Gamma_{\phi^\pm H^\mp S}(p_W, k_\mp, k_S) = 0 , \tag{42}$$

$$\mathrm{i}p_{Z,\mu} \Gamma^\mu_{ZA^0 S}(p_Z, k_A, k_S)$$
$$+ m_Z \Gamma_{\phi^0 A^0 S}(p_W, k_A, k_S) = 0 . \tag{43}$$

Again, these WIs can be checked using the explicit form of the vertices (40). In the context of the flip formalism, the trivial WIs (43) imply the absence of the gauge flips
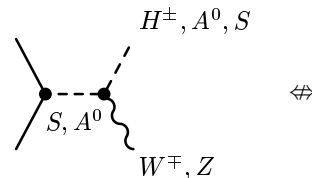
$$
,
\quad . \tag{44}
$$

Thus internal neutral Higgs bosons do not appear in the gauge flips, and therefore can be treated exactly like the SM Higgs in a non-linear parameterization. This suggests the validity of the naive ET and the gauge parameter independence of the propagators of the neutral scalars, like in the NL-SM. On a formal level, similarly to the discussion in Sects. 3 and 4, these properties are indeed a consequence of the trivial BRST transformations.

The gauge parameter dependence of the self-energies of the neutral Higgs bosons is governed by an identity of the same form as (24), also for mixed two point functions:

$$
\partial_\xi \Gamma_{H_i^0 H_j^0} = \Lambda_{\phi^0 H_i^0} \Gamma_{\phi^0 H_j^0} + \Lambda_{Z H_i^0}^\mu \Gamma_{Z H_j^0, \mu} + (i \leftrightarrow j), \tag{45}
$$

where $H_i^0 \in \{H, S, A^0\}$. Thus the self-energies can become gauge parameter dependent only if gauge–Higgs mixing occurs. This can affect the CP-odd scalar A^0 already on the one loop level. In contrast, mixing of CP-even and CP-odd Higgs bosons will induce no further gauge parameter dependence since no mixed two point functions like Γ_{HA} appear on the right hand side of (45).

5.3 Charged Higgs bosons

While the situation for neutral Higgs bosons resembles that in the NL-SM, the non-trivial BRST transformation of the charged Higgs bosons (36) implies that they have to be treated similar as in a linear parameterization. The BRST transformation (36) implies a non-trivial tree-level STI for the ZH^-H^+ vertex:

$$
\mathrm{i} p_{Z,\mu} \Gamma_{Z H^+ H^-}^\mu (p_Z, k_+, k_-) + m_Z \Gamma_{\phi_0 H^+ H^-}(p_Z, k_+, k_-)
$$
$$
= \mathrm{i} \frac{g}{\cos\theta_W} \sin^2\theta_W \left[D_{H^+ H^-}^{-1}(k_+) - D_{H^+ H^-}^{-1}(k_-) \right]. \tag{46}
$$

The STI for the $\gamma H^+ H^-$ vertex is similar. One contribution to the $Z H^+ H^-$ vertex arises from the operator $\mathcal{L}_{\mathcal{H}U}$:

$$
\mathcal{O}_{Z H^+ H^-}^{\mathcal{H}U} := \frac{\mathrm{i}}{v} \left(\partial^\mu \phi_0 + m_Z Z^\mu \right) \left(H^+ \overset{\leftrightarrow}{\partial_\mu} H^- \right). \tag{47a}
$$

The corresponding Feynman rule arising from this operator satisfies a trivial tree-level WI, i.e. it does not contribute to the right hand side of (46). This reflects the separate gauge invariance of the operator $\mathcal{L}_{\mathcal{H}U}$ that can appear with an arbitrary coefficient $\lambda_{\mathcal{H}U}$. However, there is another contribution to the $Z H^+ H^-$ vertex from the kinetic term of the Higgs bosons $\mathcal{L}_{\mathcal{H}}$, so its coefficient is fixed in agreement with the STI (46):

$$
\mathcal{O}_{V H^+ H^-}^{\mathcal{H}} := \mathrm{i} \left(H^+ \overset{\leftrightarrow}{\partial_\mu} H^- \right) \left(e A^\mu - \frac{g}{\cos\theta_W} \sin^2\theta_W Z^\mu \right). \tag{47b}
$$

Because of the non-trivial STI (46), diagrams with internal charged Higgs bosons must be included in the gauge flips. For example, the flips for sub-amplitudes including 2-fermion, charged Higgs bosons and neutral gauge bosons are given by

$$
\tilde{G}_{4,1H^\pm 2F}
$$

$$
= \left\{ \quad , \quad , \quad \right\} \tag{48}
$$

both in the linear and the non-linear parameterization.

As a consequence, there is a flip connecting self-energies and vertex corrections, as can be seen in the example of the coupling of a charged Higgs boson to b and t quarks:

$$
\Leftrightarrow \tag{49}
$$

Also, there are flips to irreducible higher loop contributions to the self-energy:

$$
\Leftrightarrow \tag{50}
$$

This indicates that charged Higgs bosons in the 2HDM cannot be resummed consistently, also in a non-linear parameterization. In agreement with this result, it is shown in Appendix A.2 that the self-energy of the charged Higgses is in general gauge parameter dependent, also without taking mixing with the gauge sector into account.

One also expects that the naive version of the ET is violated for intermediate charged Higgs bosons, i.e. for a finite width of the charged Higgs the gauge boson amplitudes and the GB amplitudes do not agree manifestly. As an explicit example, consider the process $Z \to t \bar{b} H^-$ that appears as subprocess for associated production of charged Higgs bosons at linear colliders. In the non-linear 2HDM, Yukawa couplings can be obtained from the operator
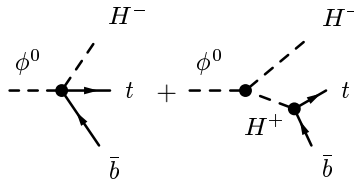
$$
\mathcal{L}_Y = -\bar{Q}_L U \mathcal{H} \left[\frac{(\lambda_t + \lambda_b)}{2} + \frac{(\lambda_t - \lambda_b)}{2} \sigma^3 \right] Q_R + \text{h.c.} \tag{51}
$$

with $m_b = \lambda_b v_1 = \lambda_b v \cos\beta$ and $m_t = \lambda_t v_2 = \lambda_t v \sin\beta$. The choice (51) corresponds to the so called type II 2HDM [1]. From an effective field theory perspective, an additional term involving only the GB matrix can be added to (51), as in the first term of (11). The effects of such non-standard Yukawa couplings on unitarity have been discussed in Sect. 3 and will not be considered in the

following. The resulting Yukawa couplings of the charged Higgs bosons are

$$\mathcal{L}_{YH^\pm} = -\frac{\sqrt{2}}{v}\left(1 + \mathrm{i}\frac{\phi^0}{v}\right)$$

$$\times \left[\bar{t}H^+\left(m_b\tan\beta\left(\frac{1+\gamma^5}{2}\right) + m_t\cot\beta\left(\frac{1-\gamma^5}{2}\right)\right)b\right]$$

$$+\,\text{h.c.} + \dots. \tag{52}$$
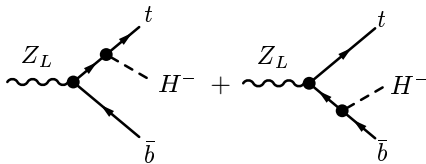
In the computation of the diagrams, we consider again the limit[4] $m_b, m_Z \to 0$. We will also include an arbitrary coefficient $\lambda_{\mathcal{H}U}$ as a factor in front of (47a). Similar to the example of top pair production in Sect. 3, the only GB diagrams with dangerous high energy behavior are the Higgs exchange and the contact diagram. Using the Feynman rules from (47a) and (52) we obtain

$$= \frac{\sqrt{2}m_t\cot\beta}{v^2}\left[\bar{t}\left(\frac{1-\gamma^5}{2}\right)b\right]\left(1 - \right.$$

$$\left. \lambda_{\mathcal{H}U}\frac{(p_{H^+}^2 - p_{H^-}^2)}{p_{H^+}^2 - m_{H^+}^2 + \mathrm{i}\,\mathrm{Im}\,\Pi_{H^+}(p_{H^+}^2)}\right) \tag{53}$$

In the case of a standard coefficient of (47b), i.e. $\lambda_{\mathcal{H}U} = 1$, the expression (53) vanishes if the external H^- is on its mass shell ($p_{H^-}^2 = m_{H^+}^2$) and the width is set to zero[5]. Since we cannot expect the ET to hold for an off-shell H^- (see below) and in order to make the cancellations among different diagrams more transparent, in the following we will keep $\lambda_{\mathcal{H}U} \neq 1$.

In the example of Sect. 3.1, we were able to decompose the amplitude into two groves that satisfied the ET by themselves. This is not the case in the present example and all diagrams for longitudinal gauge bosons are needed to reproduce the GB amplitude (53), as we will show now. The diagrams with internal fermions give an additional term compared to the contact term in (53):
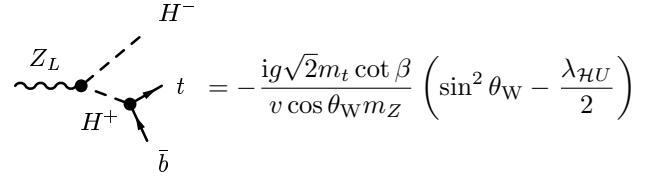
---

[4] For simplicity, we assume in addition $m_b\tan\beta \ll m_t$ but this is not essential to our argument.

[5] Actually, this could have been anticipated since in a linear parameterization the coupling of a GB to two particles with the same mass vanishes (see e.g. the second reference in [23]) so no diagram of the form (53) appears at all.

$$= -\frac{\mathrm{i}g\sqrt{2}m_t\cot\beta}{v\cos\theta_{\mathrm{W}}m_Z}\left(\frac{1}{2} - \sin^2\theta_{\mathrm{W}}\right)\left[\bar{t}\left(\frac{1-\gamma^5}{2}\right)b\right] + \dots \tag{54}$$

where we have suppressed the terms corresponding to the GB diagrams with internal fermion lines we have omitted above. The Higgs exchange diagram gives, using the Feynman rules obtained from both operators (47)

$$\left[\bar{t}\left(\frac{1-\gamma^5}{2}\right)b\right]\frac{(p_{H^+}^2 - p_{H^-}^2)}{p_{H^+}^2 - m_{H^+}^2 + \mathrm{i}\,\mathrm{Im}\,\Pi_{H^+}(p_{H^+}^2)} \tag{55}$$

Only for a vanishing width and H^- on the mass shell, the terms proportional to $\sin^2\theta_{\mathrm{W}}$ cancel between (54) and (55) and the GB amplitude (53) is reproduced (up to a phase). Therefore the situation for the charged Higgs in the non-linear parameterization is similar to a linear parameterization, as described in Sect. 3.1, and the naive version of the ET is not satisfied when a Dyson summation of the charged Higgs propagator is performed. Also since the external charged Higgs must be on the mass shell, an ET for off-shell Higgs bosons (17) is not valid for the charged Higgs bosons. These results therefore confirm the expectation of the flip formalism.

6 Summary and outlook

Motivated by the structure of gauge invariant classes of tree diagrams in non-linear parameterizations of the scalar sector [23] and the observations of [14] concerning effects of the Higgs width on the Goldstone boson equivalence theorem, we have revisited the properties of the Higgs resonance in non-linear parameterizations. As we have demonstrated for the non-linear parameterizations of both the minimal standard model and a 2-Higgs doublet model, the Dyson summation of propagators of neutral Higgs bosons can be performed without violating gauge parameter independence and Ward identities. Although in non-linear parameterizations care must be taken not to violate bounds from tree unitarity, a simple unitarity restoring expression for the Higgs propagator [15] can be used without violating the naive equivalence theorem, in contrast to linear parameterizations.

Furthermore, the full Higgs propagator has been shown to be gauge parameter independent. For the Higgs self-energy this holds only in the absence of CP-violating mixing with the gauge sector. These results are consistent with the conjectured extension of the "gauge flip" formalism to loop diagrams [22].

For charged Higgs bosons in a 2-Higgs doublet model, gauge flips exist that connect resummed self-energy diagrams to irreducible higher order contributions to the self-energy or to vertex corrections and a Dyson summation

is not compatible with gauge invariance. The violation of the naive equivalence theorem has been demonstrated for an explicit example.

The Higgs resonance has served as a first example of the application of the flip formalism to loop diagrams in a case where independent methods are available to verify the results. As mentioned in Sect. 2, a second example where the flips reproduce results established by different methods is the fermion-loop scheme [8]. A formal proof of the gauge flip formalism for one loop diagrams in linear parameterizations and applications to one loop SM processes with up to 4 fermions in the final state will be given elsewhere [24, 25]. We hope the formalism will prove useful in situations where direct proofs are difficult to achieve.

A Slavnov-Taylor and Nielsen identities

In this appendix we give some technical details on formulae used in the main text and set up our notation for the functional identities resulting from gauge invariance.

A.1 Zinn-Justin identity and STIs

The derivation of STIs for irreducible vertices uses the Zinn-Justin identity

$$\sum_\Psi \int \mathrm{d}^4 x \frac{\delta\Gamma}{\delta\Psi^\star}\frac{\delta\Gamma}{\delta\Psi} + B_a\frac{\delta\Gamma}{\delta\bar{c}_a} = 0\,, \qquad (56)$$

where the Ψ summarize all fields in the theory and the $\Psi^\star$ are the sources of the BRST transformations included in the effective action $\Gamma = \Gamma_0 + \sum_\Psi \int \mathrm{d}^4 x \,\mathrm{tr}[\Psi^\star(\delta_{\mathrm{BRS}}\Psi)]$. We will use the notation $\Gamma_{\Psi_1\ldots\Psi_n} = \frac{\delta^n\Gamma}{\delta\Psi_1\ldots\delta\Psi_n}|_{\Psi=0}$ for the irreducible vertex functions. From (56) we obtain the general STI for the HVV and the $H\phi V$ vertices by taking a derivative with respect to a ghost, a Higgs field and a gauge boson or a GB:

$$-\sum_{\Psi=V_a,\phi_a} \Gamma_{c_a\Psi^\star}\Gamma_{\Psi V_b H} \qquad (57a)$$

$$= \sum_{\Psi=V_a,\phi_a,H}(\Gamma_{c_a\Psi^\star V_b}\Gamma_{\Psi H} + \Gamma_{c_a\Psi^\star H}\Gamma_{\Psi V_b}) + \frac{\mathrm{i}}{\xi}p_b^\nu\Gamma_{c_a\bar{c}_b H}$$

$$-\sum_{\Psi=V_a,\phi_a} \Gamma_{c_a\Psi^\star}\Gamma_{\Psi\phi_b H} \qquad (57b)$$

$$= \sum_{\Psi=V_a,\phi_a,H}(\Gamma_{c_a\Psi^\star\phi_b}\Gamma_{\Psi H} + \Gamma_{c_a\Psi^\star H}\Gamma_{\Psi\phi_b}) + m_{W_a}\Gamma_{c_a\bar{c}_b H}\,,$$

where we have used the equation of motion for the auxiliary field B. For a non-linear transformation law of the GBs,

there are additional contributions of the form $\Gamma_{c\phi^\star\phi\phi}\Gamma_H$ that vanish in the absence of tadpoles. In this case both relations (57) are valid both for the linear and non-linear parameterizations. In the non-linear parameterization the Higgs drops out of the sums on the right hand side due to its trivial BRST transformation so no Higgs two point functions appear in these STIs. In higher orders, the Higgs ghost couplings $\Gamma_{c_a\bar{c}_b H}$ and the vertex functions $\Gamma_{c_a\phi_b^\star H}$ and $\Gamma_{c_a V_b^\star H}$ can be generated radiatively, but they are absent on tree level in the non-linear parameterization. Using the linear terms in the BRST transformations of the gauge bosons $\delta_{\mathrm{BRST}}V_a = \partial_\mu c_a + \ldots$ and GBs $\delta_{\mathrm{BRST}}\phi_a = -m_{V_a}c_a + \ldots$ we thus arrive at the simple tree-level WIs (5).

A.2 Gauge parameter dependence of Green's functions and irreducible vertices

An identity for the gauge parameter dependence of irreducible vertices can be derived using an extended BRST symmetry [31]. For simplicity, we suppress the indices distinguishing the gauge bosons in the following. Introducing an auxiliary Grassmann variable χ allows one to give the gauge parameters themselves a transformation law:

$$\delta_{\mathrm{BRST}}\,\xi = \chi\,, \quad \delta_{\mathrm{BRST}}\,\chi = 0\,. \qquad (58)$$

In the usual BRST formalism, the gauge fixing and ghost Lagrangian is a BRST exact operator, i.e. it can be written as a BRST transformation of a functional Θ of ghost number (-1). In the extended BRST formalism we get an additional contribution from the transformation of ξ so that

$$\delta_{\mathrm{BRST}}\Theta = \mathcal{L}_{GF} + \mathcal{L}_{FP} + \mathcal{L}_\chi\,, \quad \text{with} \quad \mathcal{L}_\chi = \chi\partial_\xi\Theta\,. \quad (59)$$

For the usual R_ξ gauge fixing $\Theta = \bar{c}\left(\partial_\mu V^\mu - \xi m_V\phi + \frac{\xi}{2}B\right)$ we find

$$\mathcal{L}_\chi = \frac{1}{2}\chi\bar{c}(B - 2m_V\phi) = -\frac{1}{2\xi}\chi\bar{c}\left(\partial_\mu V^\mu + \xi m_V\phi\right)\,, \quad (60)$$

where in the last step we have used the equation of motion for the auxiliary field B.

The Zinn-Justin identity resulting from the extended BRST transformation is given by

$$\sum_\Psi \int \mathrm{d}^4 x \frac{\delta\Gamma}{\delta\Psi^\star}\frac{\delta\Gamma}{\delta\Psi} + B\frac{\delta\Gamma}{\delta\bar{c}} + \chi\partial_\xi\Gamma = 0\,. \qquad (61)$$

Taking the derivative with respect to χ, taking the fermionic character into account, one obtains the so called "Nielsen identity" [31]

$$\partial_\xi\Gamma = \sum_\Psi \int \mathrm{d}^4 x \frac{\delta\Gamma}{\delta\Psi^\star}\frac{\delta\Gamma_\chi}{\delta\Psi} + \frac{\delta\Gamma_\chi}{\delta\Psi^\star}\frac{\delta\Gamma}{\delta\Psi} + B\frac{\delta\Gamma_\chi}{\delta\bar{c}}\,, \quad (62)$$

where we have introduced the notation $\Gamma_\chi = \partial_\chi\Gamma|_{\chi=0}$. The vertices involving insertions of χ can be evaluated using the lagrangian (60). The renormalization conditions of physical parameters have to be chosen appropriately so the Nielsen identity (62) is not deformed in higher orders [32].

As discussed in [32], imposing the vanishing of the Higgs tadpole as renormalization condition implies $\Gamma_{\chi H^*} = \Gamma_{\chi\phi^{0*}} = 0$. Taking the derivative of the Nielsen identity (62) with respect to Z we get a relation connecting these functions to $\Gamma_{\chi Z^*}$ so it is also constrained to vanish:

$$\Gamma_{\chi Z^*}\Gamma_{ZZ} + \Gamma_{\chi\phi^{0*}}\Gamma_{\phi^0 Z} + \Gamma_{\chi H^*}\Gamma_{HZ} = 0. \qquad (63)$$

In the non-linear parameterization, $\delta_{\mathrm{BRST}}H = 0$ implies that there are no vertex functions involving H^*, and one obtains for the self-energies of the neutral sector

$$\partial_\xi\Gamma_{H\Phi} = \Gamma_{\chi\phi^{0*}H}\Gamma_{\phi^0\Phi} + \Gamma^\mu_{\chi Z^* H}\Gamma_{Z\Phi,\mu}$$
$$+ \Gamma_{\chi\phi^{0*}\Phi}\Gamma_{\phi^0 H} + \Gamma^\mu_{\chi Z^*\Phi}\Gamma_{ZH,\mu}, \qquad (64)$$

with $\Phi \in \{H, \phi^0, Z\}$. For the Higgs self-energy we obtain (24) with $\Lambda_{\Phi H} = \Gamma_{\chi\Phi^* H}$. The important thing is the absence of terms involving the Higgs two point function on the right hand side, while the mixing of the Higgs is due only to CP-violation. Therefore in the absence of CP-violation one has $\partial_\xi\Gamma_{HH} = 0$. In the 2HDM, a similar identity holds for all neutral Higgs bosons (including the CP-odd) since they transform trivially. For the self-energy of the charged Higgs boson one has instead

$$\partial_\xi\Gamma_{H^+ H^-} = \Gamma_{\chi H^{+*}H^-}\Gamma_{H^- H^+} + \Gamma_{\chi\phi^{+*}H^-}\Gamma_{\phi^- H^+}$$
$$+ \Gamma_{\chi W^{+*}H^-}\Gamma_{W_L^- H^+} + (+ \leftrightarrow -). \qquad (65)$$

Here the self-energy itself appears on the right hand side and it will in general be gauge parameter dependent.

The identity governing the gauge parameter dependence of Green's functions can be derived from the extended Zinn-Justin identity (61) by Legendre transformation [31] or directly from the path integral representation of Green's functions. In operator language, it is given by

$$\partial_\xi\langle 0\,|T\,[\Psi_1\ldots\Psi_n]|\,0\rangle = \qquad (66)$$
$$-\sum_{\Psi_i}\pm\left\langle 0\left|T\left[\left(\mathrm{i}\int\mathrm{d}^4x\,\partial_\xi\Theta\right)\Psi_1\ldots\delta_{\mathrm{BRST}}\Psi_i\ldots\Psi_n\right]\right|0\right\rangle,$$

where the signs arise for fermionic fields anticommuting with the BRST transformation. Applying the LSZ formula to a given field in the Green's function, the contribution from the BRST transformed fields factorizes and can be absorbed in the wave function renormalization so (66) is also valid if the external vacuum states are replaced by physical $|\mathrm{in}\rangle$ and $|\mathrm{out}\rangle$ states.

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